TOPOLOGY I - SEMESTRAL EXAM

Time : 3 hours Max. Marks : 60

Answer all questions. You may use results proved in class after correctly quoting them. Any other claim must be accompanied by a proof.

(1) Construct a homeomorphism from the punctured plane $\mathbb{R}^2 - \{(0,0)\}$ onto the infinite cylinder $S^1 \times \mathbb{R}$. [8]

(2) Define the term *path connected.* Let $X = \{(x,y) : xy > 1\}$ have the subspace topology of $\mathbb{R}^2$. Is X path connected? Justify. [2+6]

(3) Define the terms : Hausdorff, compact, normal? Prove that a compact Haudorff space is normal. [3+5=8]

(4) Consider $[0,1]$ as an ordered set with the order induced from the usual order on $\mathbb{R}$ and let $X = [0,1] \times [0,1]$ have the dictionary order topology. Decide, with proper justifications, whether or not X has the following properties : least upper bound property, connected, locally connected, path connected, locally path connected, compact, locally compact, first countable, second countable and normal. [2 x 10 =20]

(5) When is a metric space complete? Show that a metric space (X,d) is complete if and only if for every nested sequence
$$A_1 \supset A_2 \supset \cdots$$
of non empty closed subsets of X such that $\mathrm{diam}(A_n) \to 0$, the intersection of the sets A_n is non empty. [1+7=8]

(6) When do you say two spaces X and Y are homotopically equivalent? Let X denote the set of integers and $Y = \{1/n\}_{n \geq 1} \cup \{0\}$ both with the subspace topology of $\mathbb{R}$. Show that X and Y are not homotopically equivalent. [2+6=8]